Method Used to Simulate Cycling Speeds and Finishing Times for the 2015 and 2016 Transcontinental Races

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This document describes the method used to simulate cycling speeds and finish times for the Transcontinental Race (TCR). The results of these simulations is reported online: http://www.murray-white.net/ultra/speed.html.

There are five forces that resist a cyclist's forward motion: air resistance, tyre rolling resistance, mechanical resistance in the drivetrain and hubs, gravitational resistance during uphills, and braking forces. The two forces that propel a cyclist forwards are the power that the rider exerts through the pedals and the gravitational force experienced on downhills. A tailwind can occasionally move a cyclist forward, but at most riding speeds tailwinds are only strong enough to reduce the negative affect of air resistance rather than creating a positive force.

How these factors determine cycling speeds using relatively simple equations is already well-established, and several websites have interfaces where these equations are used to predict cycling speeds based on user-controllable input variables (e.g., http://www.gribble.org/cycling/power_v_speed.html). My model goes beyond most others by reading altitude and distance data for an entire route and predicting the average speed for a rider with a given power profile and many other manipulable variables.

I describe below the equations used by the model to predict cycling speeds, I then describe what information was extracted from a complete route and how average speeds and total riding times were computed. I finish by explaining what additional equations and assumptions were used to allow the model to incorporate the more subtle effects like how elevation and wind are expected to affect the predicted speeds.

Forces Acting on a Cyclist

The force exerted by air resistance, also called aerodynamic drag (denoted as F_{air} and measured in N) is a particularly important factor. It increases based on air density (ρ, or rho, measured in kg/m^3), which is explained more below in the section on the effects of elevation. It also increases based on the frontal surface area of the cyclist, bicycle, and all equipment (A, measured in m^2), and the drag coefficient (C_d, which has no units), which is a measure of how aerodynamic something's shape is (e.g., brick-shaped vs. wing-shaped). The surface area and drag coefficient are often treated as one measure that is written as C_dA. Values used for this variable are shown in Table 1 below; the effects of manipulating this are included in the online results. Finally, the force of air resistance increases with the square of the cyclist's speed into the wind (aka airspeed, v_a, measured in m/s). The exact equation is:

$$F_{air} = 0.5 \cdot C_d \cdot A \cdot \rho \cdot v_a^2 \qquad (1)$$

Note that windspeed is used in Equation 1 instead of groundspeed. The assumptions concerning how windspeed sometimes differs from groundspeed are discussed in the section on the Effects of Wind below.

The force exerted by tyre/tire rolling resistance (F_{roll}, measured in N) increases based on the total mass of the cyclist, bike, and equipment (m, measured in kg) and the rolling resistance coefficient of the tires used (C_{rr}, which has no units). The default value for the total mass was 85 kg; the effects of manipulating this are included in the online results. Values for the coefficient of rolling resistance were based on those given at http://www.bicyclerollingresistance.com; the effects of manipulating this are also included in the online results. The gravitational constant (g = 9.8067 m/s^2) is also needed to transform mass into weight. It is actually the *normal* weight which is needed, which is the force exerted *perpendicular* to the road surface, which varies based on the gradient of the road (G, measured as % of height gained for a certain distance ridden), which is the reason for the trigonometry in the following equation:

$$F_{roll} = C_{rr} \cdot \cos[\ \arctan(\ G / 100\ )\ ] \cdot m \cdot g \qquad (2)$$

The force exerted by gravity (F_{grav}, measured in N) obviously increases based on the total mass and the gradient of the road. The specific gradient values used in the modeling are shown in Table 1, which are based on the mean observed values in each range for each route. Again, the gravitational constant is needed to convert mass to weight:

$$F_{grav} = \sin[\ \arctan(\ G / 100\ )\ ] \cdot m \cdot g \qquad (3)$$

These three measures of force need to be converted to measures of power (P_{air}, P_{roll}, and P_{grav}, each measured in W). Power is the total work done to overcome a force during one second. The distance covered in one second is given by the groundspeed (v_g, measured in m/s), which is hereafter referred to more simply as speed. Force is therefore converted to power using the following equations:

$$P_{air} = F_{air} \cdot v_g \qquad (4)$$

$$P_{roll} = F_{roll} \cdot v_g \qquad (5)$$

$$P_{grav} = F_{grav} \cdot v_g \qquad (6)$$

The total resisting power (P_{resist}, measured in W) is simply the sum of these three:

$$P_{resist} = P_{air} + P_{roll} + P_{grav} \qquad (7)$$

To continue moving at a constant speed, the power exerted by the wheels must equal the resisting power, so:

$$P_{wheel} = P_{resist} \qquad (8)$$

Unfortunately, not all of the power put into the pedals by the cyclist (P_{legs}, measured in W) reaches the wheel due to mechanical resistance and drivetrain losses (L_{dt}, measured in %).

Exactly what is included in drivetrain losses is described more below, and the effects of manipulating this are included in the online results. Therefore, the power exerted by the legs and power transmitted by the wheels are related using the following equation:

$$P_{wheel} = P_{legs} \cdot [\ 1 - (L_{dt} / 100)\ ] \quad (9)$$

The power lost to mechanical resistance and drivetrain losses (P_{mech}, measured in W) can then be computed as:

$$P_{mech} = P_{legs} - P_{wheel} \quad (10)$$

The power exerted by the legs is assumed to vary based on the gradient of the road. Two arrays of values were used, one estimated for an average-strength TCR rider and one estimated for a strong TCR rider, which are shown in Table 1.

Table 1: Air resistance, power, and braking values for each type of simulated rider at each gradient range.

Gradient Range	Gradient Values Used, G, 2015 Route	Gradient Values Used, G, 2016 Route	C_dA (m^2)	P_{legs} (W), Average rider	P_{legs} (W), Strong rider	Braking (%)
< -7%	-8.9%	-9.1%	0.37	0	0	50
-7% → -4%	-5.3%	-5.3%	0.37	0	40	25
-4% → -1%	-2.1%	-2.3%	0.37	40	80	0
-1% → 1%	0.0%	0.0%	0.4	130	170	-
1% → 4%	2.1%	2.2%	0.43	160	200	-
4% → 7%	5.3%	5.3%	0.43	180	220	-
> 7%	9.0%	9.0%	0.43	200	240	-

The equations above are presented as if speed is a known variable and forces and power values are determined based on this and the other variables. In fact, the power generated by the cyclist was treated as the known variable, and then the speed that would result from this was then computed. Because the power of air resistance increases as a *cubic* function of speed, a cubic equation must be solved. This was done using an iterative method of trying an approximate value for speed and then adjusting the value based on the direction of the error until the error fell below a predefined threshold.

Many people intuitively expect that even though more power tends to be exerted on steeper climbs than on gentler climbs or flat sections, this difference may diminish when doing longer climbs. I spent some time looking for such a difference in observed data, but I found almost no evidence for it regardless of the definition of a "longer" climb. Any difference that does exist seems to be overshadowed by the noise in the data, so either there is no such effect or it is so small that ignoring it won't significantly reduce the accuracy of the model, so I have not included this complication.

Braking Adjustment

This simple model does a very good job of predicting observed speeds based on observed power on flat sections and on climbs, with most differences being less than 0.5 km/h when using reasonably long routes. Unfortunately, average speeds on descents are predicted less well due to the observed speeds tending to be lower than those that are predicted. This is partly because corners require the brakes to be used and also because the model assumes that a "terminal velocity" is achieved instantaneously for each gradient range. Such a velocity will be reached far more quickly on climbs and flat sections than during descents, when more time is needed to accelerate up to such speeds, so the lack of accuracy in these conditions is not surprising.

The simplest way to make the model predict more realistic speeds during descents is to apply a braking factor that removes a certain percentage of the force being applied by gravity on the descents. This method causes the effect of braking to be somewhat overestimated, but the only alternative would be to develop a completely different modeling approach that incorporates principles of Newtonian physics, momentum, and acceleration. The predictive accuracy and efficacy of such a model would probably not be much higher than is the case when using the more basic approach described here.

The braking-adjusted force exerted by gravity (F'_{grav}, measured in N) is simply the regular force exerted by gravity reduced by the braking value (B, measured in %):

$$F'_{\mathrm{grav}} = F_{\mathrm{grav}} \cdot [\ 1 - (\mathrm{B} / 100)\ ] \qquad (11)$$

F'_{grav} is then used in Equation 6 instead of F_{grav}. The resisting force of braking (F_{brake}, measured in N) can be computed as:

$$F_{\mathrm{brake}} = F_{grav} - F'_{grav} \qquad (12)$$

Values of 50% braking for steep descents, 25% braking for moderate descents, and 0% for gentle descents are used because they provide a reasonable fit to a variety of observed data. These values are kept constant for all simulations. This is not a real reflection of the importance of braking in the real-world, so braking forces are not discussed in the results.

Mechanical Resistance

There are two components of mechanical resistance that slow down a cyclist. The first is the wheel hubs. In most models, this is combined or measured together with tire rolling resistance, but here it is treated separately so that the effect of using a dynamo hub can be investigated. Separate coefficients of hub resistance for the front and rear hubs (C_{hrf} and C_{hrr}, respectively, which have no units) are combined with the speed to compute the power lost due to hub resistance (P_{hubs}, measured in W):

$$P_{\mathrm{hubs}} = (\ C_{\mathrm{hrf}} + C_{\mathrm{hrr}}\ ) \cdot v_{\mathrm{g}} \qquad (13)$$

The coefficient of hub resistance for a hub with standard steel bearings is assumed to be 0.00008 (which is used by default for the rear hub) and for ceramic bearings this is halved, so

0.00004; the effect of manipulating which types of hubs are used is included in the online results.

When a good-quality dynamo hub is used, the coefficient of hub resistance for the front hub is determined by the percentage of time that the hub is in use to power devices (*D*, measured in %). When a dynamo hub is not being used to power any lights or devices, the coefficient of hub resistance is 0.0002 (C_{doff}, which has no units) and when it is powering something then it is 0.001 (C_{don}, which has no units). Published values for resistance caused by dynamo hubs is typically given as a graph showing watts consumed across a range of speeds. These coefficients were chosen so that the values shown in those graphs would be approximately replicated. The default condition is that the bicycle is equipped with a dynamo hub which is in use 25% of the time. The effect of manipulating whether a dynamo hub was used and what percentage of time is included in the online results.

$$C_{hrf} = (D / 100) \cdot C_{don} + ([100 - D] / 100) \cdot C_{doff} \quad (14)$$

Equation 7, which computes the total resisting power, is therefore updated to also include hub resistance:

$$P_{resist} = P_{air} + P_{roll} + P_{grav} + P_{hubs} \quad (7b)$$

The rest of the mechanical resistance comes from the drivetrain. Above, these losses were summarized as the factor L_{dt} in Equation 9. The drivetrain losses are assumed to consist of losses caused by the pedals, bottom bracket, chain, and derailleur pulleys (L_p, L_{bb}, L_c, and L_{dp}, respectively, measured in %):

$$L_{dt} = L_p + L_{bb} + L_c + L_{dp} \quad (15)$$

Research published by Friction Facts (https://www.friction-facts.com/) was used to determine the rough values for these variables. L_p was fixed at 0.1% for all pedals. L_{bb} can be 0.2% for a high-quality bottom bracket or 0.5% for a standard-quality model; 0.5% is the default value. L_c is 3% for a regular chain, but an extra 1% is added for an old chain and 1% for a dry chain with no oil; 3% is the default value. L_{dp} is 0.5%, 1.0%, 1.5%, or 2.0% depending on whether oversized pulleys with ceramic bearings are used, standard-sized pulleys with ceramic bearings, standard-sized pulleys with steel bearings, or standard-sized pulleys with bushings are used, respectively; 1.5% is the default value. The default value of the total drivetrain losses, L_{dt}, is therefore 5.1%. The effect of manipulating each of these components is included in the online results.

Route Data

Route data for the 2015 and 2016 Transcontinental Races were entered into the model. The routes were divided into segments that were about 500 to 1000 km long, generally corresponding to where checkpoints were located or where major geographic borders were crossed.

Table 2: Data for each segment of the 2016 Transcontinental Race route based on each gradient range.

Gradient Range	Start – Puy de Dôme			Puy de Dôme – Interlaken			Interlaken – Slovenia			Slovenia – Montenegro			Montenegro – Finish			Totals		
	Dist' (km)	Mean Elev' (m)	Pred'd Speed (km/h)	Dist' (km)	Mean Elev (m)	Pred'd Speed (km/h)	Dist' (km)	Mean Elev (m)	Pred'd Speed (km/h)	Dist' (km)	Mean Elev (m)	Pred'd Speed (km/h)	Dist' (km)	Mean Elev (m)	Pred'd Speed (km/h)	Dist' (km)	Mean Elev (m)	Pred'd Speed (km/h)
< -7%	9	449	41.7	17	554	41.9	102	1375	43.3	67	744	42.2	36	875	42.4	232	1013	42.6
-7% → -4%	34	309	38.7	37	517	39.0	68	1236	40.2	69	652	39.2	51	582	39.1	259	727	39.3
-4% → -1%	157	206	31.8	108	464	32.1	133	1036	32.6	174	606	32.2	222	424	32.0	793	529	32.1
-1% → 1%	302	200	26.0	182	444	26.0	144	966	26.0	235	560	26.0	501	264	26.0	1364	399	26.0
1% → 4%	169	214	18.6	113	478	18.5	118	1060	18.2	166	612	18.4	197	420	18.6	763	523	18.5
4% → 7%	31	343	11.9	36	524	11.8	66	1272	11.3	72	717	11.7	52	654	11.8	257	775	11.7
> 7%	11	449	8.4	18	552	8.4	103	1344	7.9	71	769	8.3	30	865	8.2	232	1008	8.1
Total / Avg	713	233	23.3	511	480	22.2	732	1193	18.8	854	645	20.6	1089	455	23.2	3899	598	21.55

Note: Dist' = distance, Elev' = elevation, Pred'd = predicted

The average elevation in every 200-metre-long piece of the route segment was compared to that of the previous such piece; this was then compared to the distance between those two pieces to obtain the gradient for that 200-metre long piece of route. The total distance covered by all pieces in each segment of the route that fell within each of seven ranges of gradients was then obtained. The mean elevation of the pieces contained in each gradient range in each segment was also computed. These values are shown in Table 2 for the 2016 route along with the predicted speeds.

The elevation data was sometimes erratic and unreliable (e.g., when the road passes through a tunnel) and so any observed gradients for a piece of route above 15% or below -15% were not included when computing the mean observed gradients within each range (shown in the second and third columns of Table 1). These pieces of route were included in the total distances recorded for each gradient range.

Effects of Elevation

As mentioned above, the mean elevation for each gradient range for each route segment was used in the model. Elevation has three separate effects on riding speeds due to atmospheric factors. Most people know that temperature decreases as altitude/elevation increases, the rate is typically 0.65ºC for every 100 metres gained. The first effect this has on cycling speeds is that tires have more rolling resistance when the temperature is lower, increasing by 1.4% for every 1ºC lost, so rolling resistance increases by about 1% for every 100 metres gained, causing cyclists to move slower for the same amount of power exerted. The air is not only cooler at higher elevations, it is also significantly less dense, the difference in density being about 0.8% for every 100 metres gained, which causes cyclists to move faster for the same amount of power exerted. Finally, there is also less oxygen available as the elevation increases, which reduces the power that a cyclist can exert, with about 0.7% power lost for every 100 metres according to the formulas given by Bassett et al. (1999). How each of these effects was incorporated into the model is described below.

Air density is measured by rho (ρ, measured in kg/m^3), and is an important component of Equation 1, the force exerted by air resistance. It is computed using the following equations:

Because the TCR is held in the middle of summer and people ride mostly during daylight hours, the temperature at sea level (T_0, measured in K) is assumed to be a constant 298.15 (which is 25ºC). The temperature (T, measured in K) at the current elevation (h, measured in m) is computed based on the temperature at sea level and the rate that temperature decreases with elevation, L (= 0.0065 K / m).

$$T = T_0 - L \cdot h \tag{16}$$

The atmospheric pressure at sea level is assumed to be constant and the standard value is used (p_0 = 101.325 kPa). The pressure (p, measured in kPa) at the current elevation is then computed based on the pressure at sea level, an adjustment for elevation, the gravitational constant g, the molar mass of dry air (M = 0.02896 kg / mol), the ideal universal gas constant (R_U = 8.315 J / [mol · K]), and L.

$$p = p_0 \cdot [\, 1 - (L \cdot h / T_0) \,]^{(g \cdot M) / (R_U \cdot L)} \quad (17)$$

The air density (ρ) can then be computed based on the temperature and pressure at the current altitude plus molar mass and the specific gas constant (R_S = 287.058 J / [kg · K]).

$$\rho = p \;/\; (\, R_S \cdot T \,) \quad (18)$$

This is the air pressure value that is used in Equation 1.

Coefficients of rolling resistance for tires are typically published assuming a temperature of 20ºC (T_{rr} = 293.15 K). Rolling resistance is known to vary according to air temperature; Tom Anhalt (http://bikeblather.blogspot.ch/) has estimated the rate of decrease of C_{rr} to be 1.36% per degree Celsius (i.e., F = 0.0136). The temperature-adjusted rolling resistance coefficient (C'_{rr}) is therefore used in Equation 2 instead of the basic value, which is computed as follows:

$$C'_{rr} = C_{rr} \cdot (\, 1 + F \cdot [T - T_{rr}] \,) \quad (19)$$

Air contains less oxygen at higher elevations, which causes the maximum power that can be sustained to be reduced. Bassett et al. (1999) measured these effects with trained athletes and gave the following cubic equation as the best-fitting function to explain how available power is reduced based on current elevation (h, this time measured in km):

$$P'_{legs} = P_{legs} \cdot (\, 0.1783 \cdot h^3 - 1.43 \cdot h^2 - 4.07 \cdot h + 100 \,) / 100 \quad (20)$$

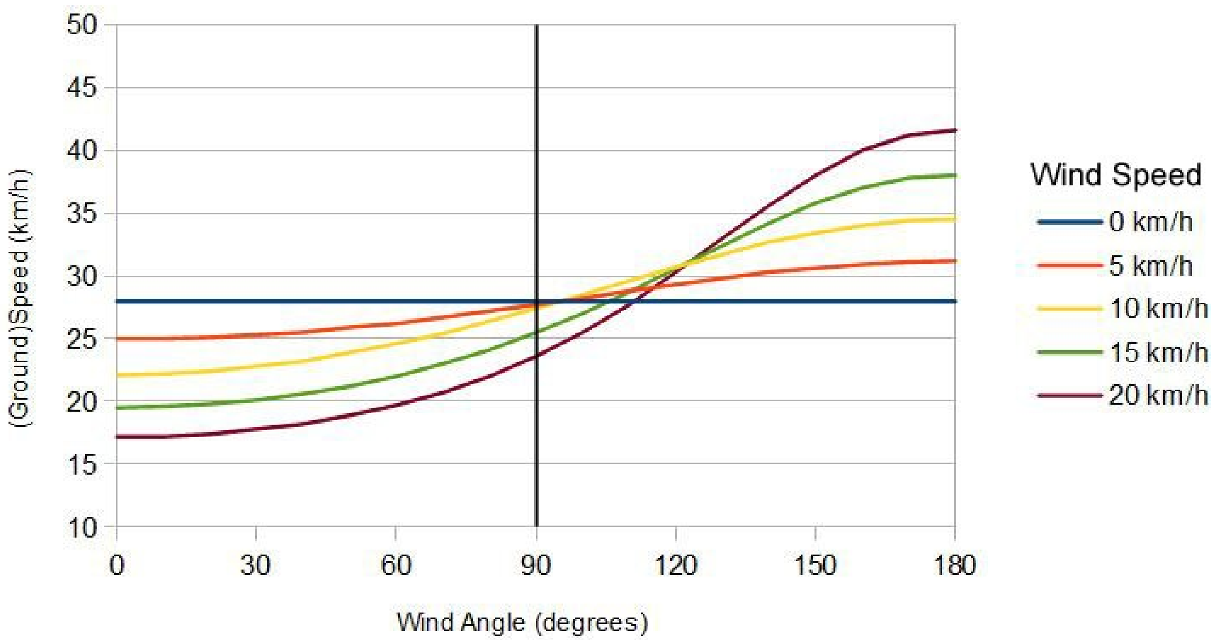
This elevation-adjusted power value was used in Equations 9 and 10 instead of the power attainable at sea level. How predicted speeds differ at different altitudes based on these three adjustments is covered in the online results.

Effects of Wind

Average wind speeds at ground level in most of Europe tend to be relatively low (e.g., see here: https://deepresource.wordpress.com/2014/12/04/european-wind-potential/), except for the regions surrounding the North Sea and on the Atlantic coast. Based on this data, I make the rough approximation that the average wind speed during the TCR is 10 km/h. Because the effect of wind is non-linear, I actually implement this by imposing winds for an equal amount of time at 0, 5, 10, 15, and 20 km/h and averaging across these scenarios. I found no solid evidence showing a consistent trend in the wind direction across Europe in the summer, so I assume that the wind is equally likely to come from any direction, and so take the average across all winds at 15 degree increments.

Figure 1 shows predicted speeds for a cyclist on a flat road who averages 150 watts and has a constant C_dA of 0.4 when experiencing varying wind speeds (shown by lines of different colors) and from various angles (shown on the *x*-axis). Some people may initially assume that if the wind is equally likely to be a headwind, tailwind, or perfect crosswind then this will be effectively the same as there being no wind. However, even a perfect crosswind at a 90° angle causes more air resistance than what exists when there is no wind, as shown by the four lines for non-zero winds being lower than that for zero wind (the blue line) in Figure 1. As a specific example, a perfect 90-degree crosswind of 20 km/h will slow down the hypothetical cyclist by more than 4 km/h (from 28 km/h to 23.6 km/h). This is because the cyclist effectively travels further through the wind in this situation than they do along the ground, see Jobst Brandt's full explanation: http://www.sheldonbrown.com/brandt/wind.html.

Figure 1: Predicted (ground) speeds for different wind angles and wind speeds on a flat road and given a constant power of 150 watts and a constant C_dA of 0.4.



A further complication is that the rider and bike tend to be aerodynamically optimized for going into a pure headwind. When there is any crosswind, the rider and bike tend to be less aerodynamic than in a pure headwind (the exception being that some wheel rims are more aerodynamic in certain crosswinds). The C_dA data estimated by Osman Isvan (http://www.jsc-journal.com/ojs/index.php?journal=JSC&page=article&op=view&path[]=168) was used as a basis for the correction of adding 0.03 to the C_dA value when the apparent wind angle (aka yaw) that the cyclist experiences is 15 to 25 degrees, and 0.05 when the apparent wind angle is greater than 25 degrees.

The following equation was used to compute air speed based on the ground speed, wind speed (v_w, measured in m/s) and wind angle (α, measured in radians):

$$v_a = \sqrt{[(v_g + v_w \cdot \cos(\alpha))^2 + (v_w \cdot \sin(\alpha))^2]} \quad (21)$$

The apparent wind angle, or yaw (β, measured in radians) was computed as follows:

$$\beta = \mathrm{acos}[(v_g + v_w \cdot \cos(\alpha)) / v_a] \quad (22)$$

As mentioned above, airspeed was used when calculating the force of air resistance:

$$F_{air} = 0.5 \cdot C_d \cdot A \cdot \rho \cdot v_a^2 \quad (1)$$

But to convert this into power, ground speed is needed in addition to the apparent wind angle:

$$P_{air} = F_{air} \cdot v_g \cdot \cos(\beta) \quad (23)$$